Let ω' and v' be, respectively, the angular velocity, and the horizontal velocity of the center of the sphere, after the first impact.

The impulsive action at the point of contact is, then, Mv'.

The change in the angular momentum being equal to the moment of the impulse,

$$\tfrac{2}{5}MR^2(\omega-\omega')=Mv'R.$$

The surfaces being perfectly rough, there is no slipping and

$$v'=R\omega'$$

$$\therefore\ \tfrac{2}{5}(\omega-\omega')=\omega',$$

$$\omega'=\tfrac{2}{7}\omega\,;$$

$$v'=\tfrac{2}{7}R\omega.$$

v'' and ω'' representing horizontal and angular velocities after second impact,

$$M(v''-v')=\text{impulsive friction},$$

$$\tfrac{2}{5}MR^2(\omega''-\omega')=-M(v''-v')R,$$

$$\tfrac{2}{5}(\omega''-\omega')=\omega'-\omega'',$$

$$\omega''=\omega'=\tfrac{2}{7}R\omega,\text{ and } v''=R\omega''=\tfrac{2}{7}R\omega.$$

The sphere moves on in an endless series of equal parabolas, with constant angular velocity and constant horizontal velocity, reaching the height a after every rebound.

Solutions of this problem were also received from Professors Zerr and Anthony. One or both of these solutions will appear in the next issue of the MONTHLY.

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

30. Proposed by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

A and B are two integers, A consisting of $2m$ figures each being 1, and B consisting of m figures each being 4. Prove that $A+B+1$ is a square.

I. Solution by H. W. DRAUGHON, Olio, Mississippi, and O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

Each of the integers is a Geometrical Series.

$A=1+10+100+$etc., to $2\ m$ terms, $=\frac{1}{9}(10^{2m}-1)$.

$B=4+40+400+$etc., to m terms, $=\frac{4}{9}(10^m-1)$.

$A+B+1=\frac{1}{9}(10^{2m}-1)+\frac{4}{9}(10^m-1)+1=\frac{1}{9}(10^{2m}+4.10^m+4)$

$=\{\frac{1}{3}(10+2)\}^2$.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas, and H. C. WILKES, Skull Run, West Virginia.**

$A=\frac{9}{16}B^2+\frac{1}{2}B$ as is shown by the following: Let $B=444$.

$\therefore\ \frac{9}{16}B^2+\frac{1}{2}B=111111$. This is true for any value of B.

Hence $A+B+1=\frac{9}{16}B^2+\frac{1}{2}B+1=\left(\frac{3B+4}{4}\right)^2=B^2$.

$\therefore\ A+B+1=(333......334)^2$, the number within the parenthesis consists of m figures. Let A_1 be an integer consisting of m figures all 1's.

Then $B^2=(333......334)^2=(B+1+A_1)^2$.

Also solved by *M. A. GRUBER* and *J. SCHEFFER*.

31. **Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.**

How many scalene triangles, of integral sides, can be formed with an altitude of 12? How many isosceles triangles?

I. **Solution by ARTEMUS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington D. C.**

1. To find right-angled triangles having one leg$=12$.

Let $x=$the required leg and $x+a=$the hypothenuse; then $(x+a)^2-x^2=2ax+a^2=12^2=144$; whence $x=\dfrac{144-a^2}{2a}$.

It is easily seen that a must be even, and that it cannot exceed 10; but as x must be integral a can only be 2, 4, 6, or 8.

Take $a=2$, then $x=35$; take $a=4$, then $x=16$; take $a=6$, then $x=9$; take $a=8$, then $x=5$. Hence there are four right-angled triangles having one leg $=12$, viz: 12, 35, 37; 12, 16, 20; 12, 9, 15; 12, 5, 13.

2. Any two right-angled triangles, p, c, a; p, b, d, can be combined in two different ways to form a scalene triangle, giving the triangles a, b, $c+d$; a, b, $c-d$. Hence the four right-angled triangles found above can be combined two and two in two different ways to form scalene triangles; therefore there are twelve such triangles which have an altitude of 12, as follows: 13, 14, 15; 20, 37, 51; 15, 20, 25; 15, 37, 44; 13, 37, 40; 13, 20, 21; 13, 15, 4; 20, 37, 19; 15, 20, 7; 15, 37, 26; 13, 37, 30; 13, 20, 11.

There can be only four isosceles triangles with integral sides having an altitude of 12, viz: 13, 13, 10; 15, 15, 18; 20, 20, 32; 37, 37, 70.

II. **Solution by A. H. BELL, Box 184, Hillsboro, Illinois.**

We evidently require to find two □ numbers whose difference shall be equal to any given number. Let $x=$the side of the lesser square, and $d=$to